15138 수치해석 Project #1

화학공학과 김성균

2017032597

Review: SVD, Eigenfaces

- SVD
  - A method to find vector basis
  - Can be applied to **general matrices**
  - Considering vector distribution

    -> **efficient**
  - Compact representation of vector space
    - Small number of **dominant** basis vectors
    - Pattern recognition, Data compression

- Eigenfaces
  - Basis vectors of the face images
  - Each pixel is an element of the face vector
  - Face Recognition (Synthesis)

    Linear combination of eigenface vectors

Introduction to the Assignment

- Singular Value Decomposition (SVD) of face images
  - Collect dataset, find eigenfaces by SVD
- Synthesis of face images using the eigenfaces
- Experiments of face recognition
- Analysis and Conclusion

Strategy

Step 1 Collect dataset (face images)

- (More images than vector dimension if possible)

Step 2 Construct data matrix A

- Find the mean vector and subtract from each face vector
- Covariance matrix = $A^T A$ (or AA^T if n > dim)

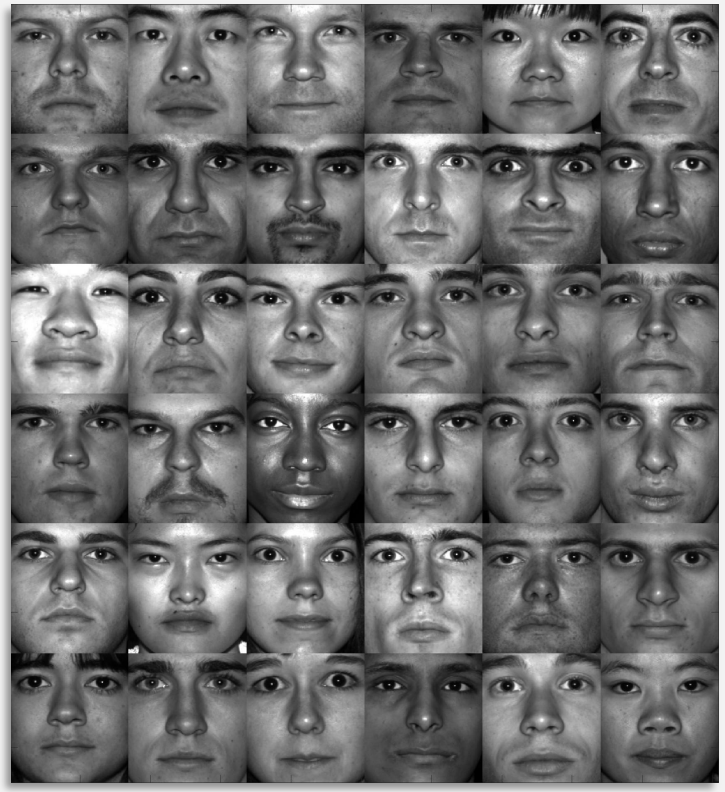
Apply SVD

- Find eigenfaces

Step 3 Reconstruct collected face images using eigenvectors

Step 4 Compare coefficients

Step 1: Collect dataset



〈 Collected face images: 36 persons shown 〉

〈 64 images of person #1 〉

- Yale_B dataset
  - Total 38 persons of different races

    (Use 36 of them for training)
  - About 64 images of each person with different lightings

    -> 2282 face vectors
  - Resolution: 192x168 (pixels)

    -> face vector: 32256-dimension
- # of images < dimension of face vector
  - Covariance matrix = $A^T A$

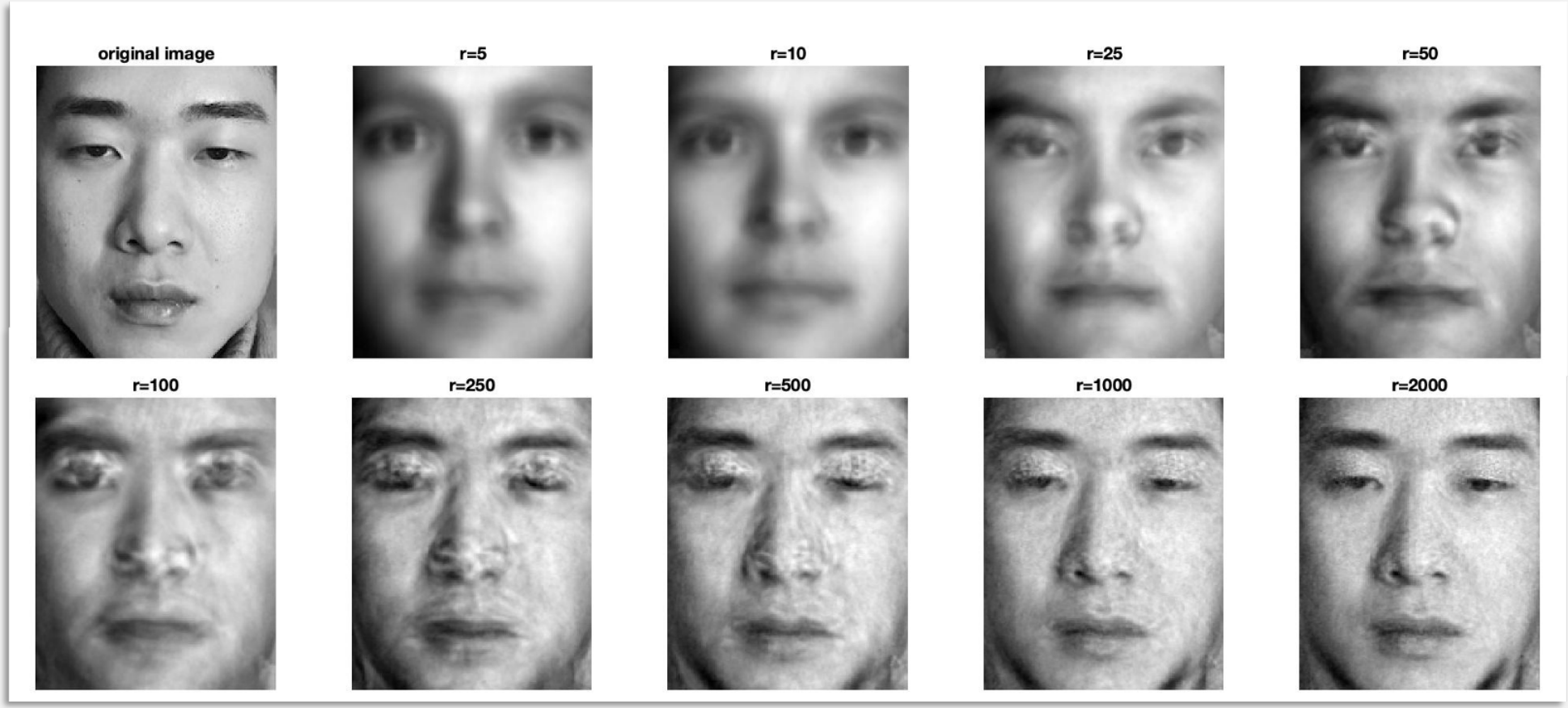
Step 2 : Construct data matrix A, Apply SVD

```
trainingFaces = faces(:,1:sum(nfaces(1:36)));
avgFace = mean(trainingFaces,2);
A = trainingFaces-avgFace*ones(1,size(trainingFaces,2));
[U,S,V] = svd(A,'econ');
```

〈 Mean vector = average face 〉

〈 The first 36 eigenfaces 〉

Step 3: Test face recognition - Reconstruction



〈 Face reconstruction 〉

- Reconstruction was successful when $r > 1000$ in general

Step 4: Test face recognition - Comparing coefficients

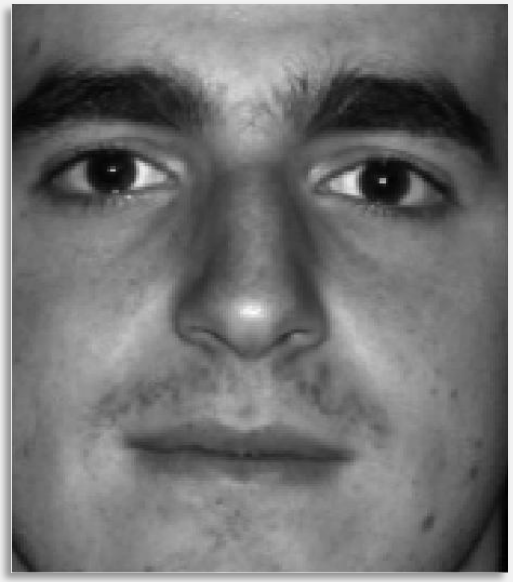
```
P1num = 37;  % person number 37
P2num = 38;  % person number 38
P1 = faces(:,1+sum(nfaces(1:P1num-1)):sum(nfaces(1:P1num)));
P2 = faces(:,1+sum(nfaces(1:P2num-1)):sum(nfaces(1:P2num)));
P1ms = P1 - avgFace*ones(1,size(P1,2)); % subtract mean vector
P2ms = P2 - avgFace*ones(1,size(P2,2));

figure;
subplot(1,2,1), imagesc(reshape(P1(:,1),n,m)); colormap gray, axis off
pbaspect ([168 192 1])
subplot(1,2,2), imagesc(reshape(P2(:,1),n,m)); colormap gray, axis off
pbaspect ([168 192 1])

% project onto eigenfaces
PCAmodes = [6 3];

PCACoordsP1 = U(:,PCAmodes)'*P1ms;
PCACoordsP2 = U(:,PCAmodes)'*P2ms;

figure;
plot(PCACoordsP1(1,:),PCACoordsP1(2,:),'kd','MarkerFaceColor','k')
axis ([-6000 6000 -6000 6000]), hold on, grid on
plot(PCACoordsP2(1,:),PCACoordsP2(2,:),'r^','MarkerFaceColor','r')

xlabel(['c ' num2str(PCAmodes(1))])
ylabel(['c ' num2str(PCAmodes(2))])
```

<- Projection of each person's each face vector onto two chosen eigenfaces

〈 Implementation: comparing coefficients 〉

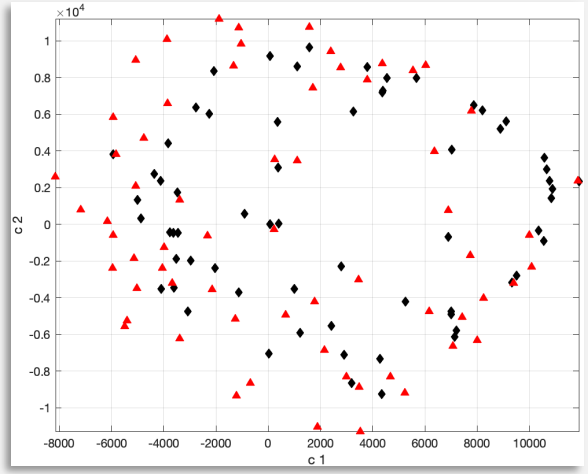
- Comparing two persons **out of** training set
- Choose two arbitrary coefficients c_i, c_j (projection onto two eigenfaces) to plot (for all 64 faces of each person)

〈 Person #37 〉

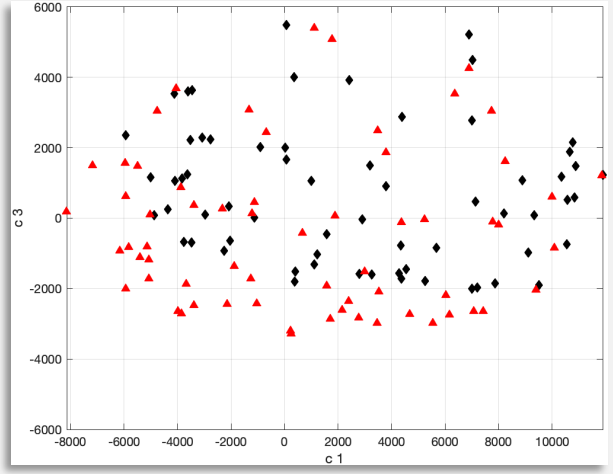


〈 Person #38 〉

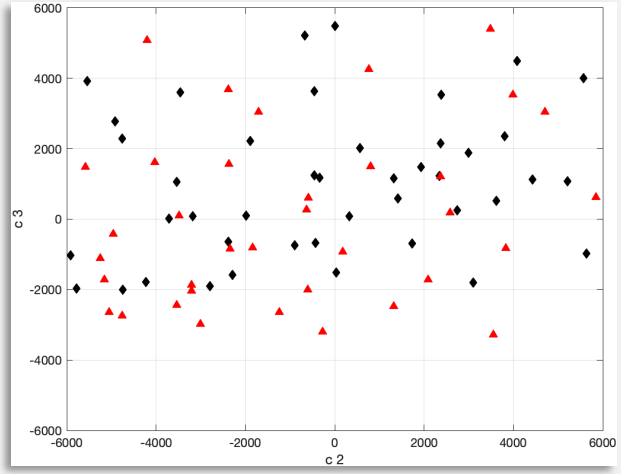
Step 4: Test face recognition - Comparing coefficients



〈 Plotting c1 & c2 〉

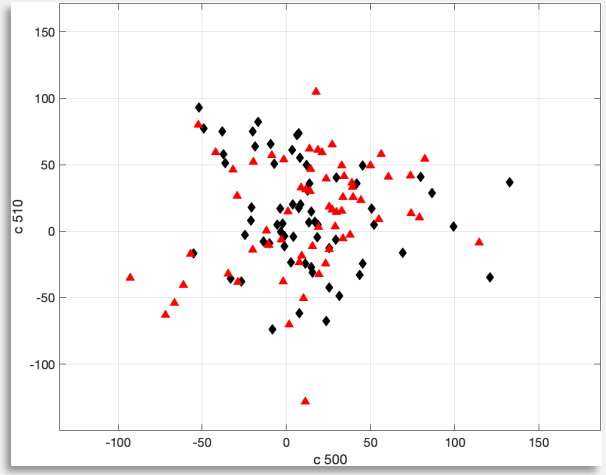


〈 Plotting c1 & c3 〉

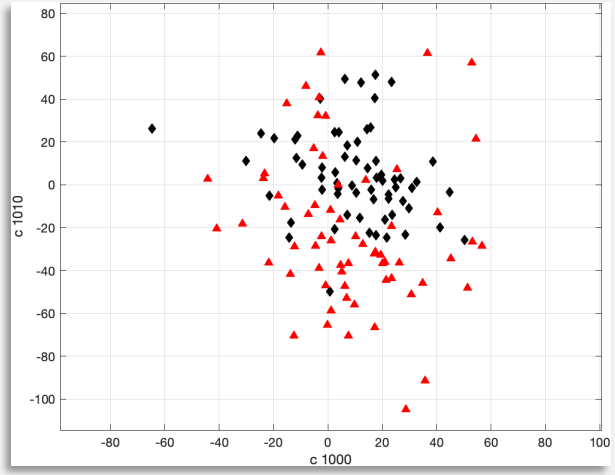


〈 Plotting c2 & c3 〉

Black: person #37
Red: person #38



〈 Plotting c500 & c510 〉



〈 Plotting c1000 & c1010 〉

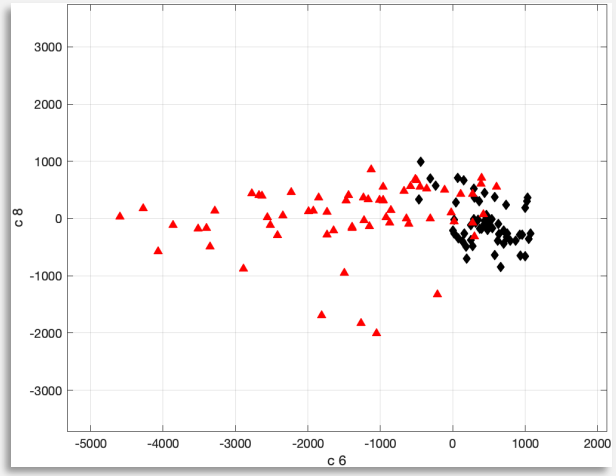
- **Unsuccessful** recognition if i and j are too small

  -〉 **commonly dominant** features in both faces

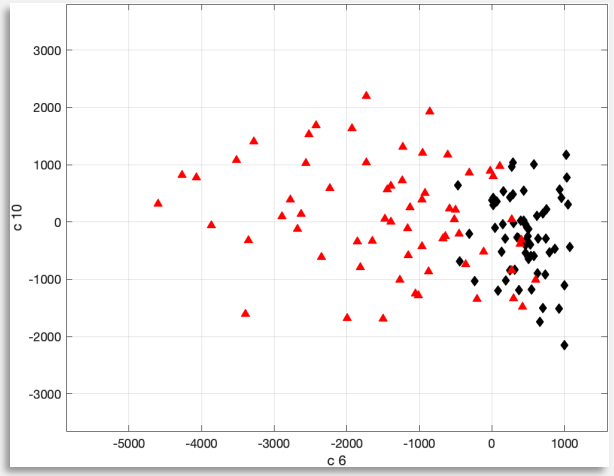
- **Unsuccessful** recognition if i and j are too big

  -〉 **commonly minor** features in both faces

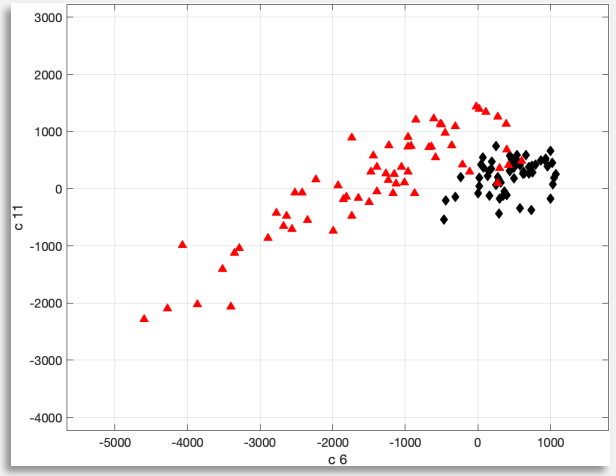
Step 4: Test face recognition - Comparing coefficients



〈 Plotting c6 & c8 〉



〈 Plotting c6 & c10 〉



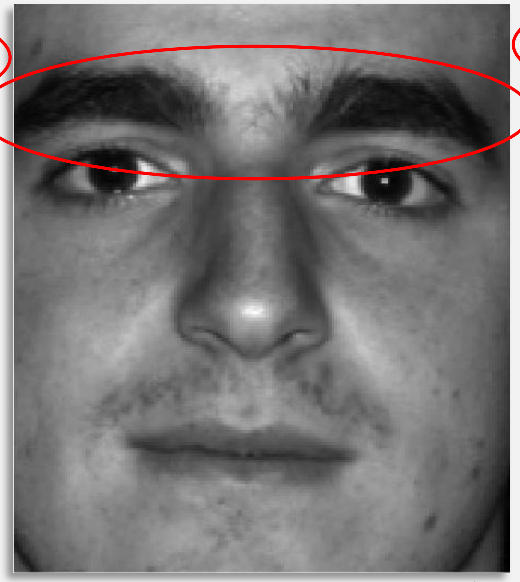
〈 Plotting c6 & c11 〉

Black: person #37
Red: person #38

- **Successful** if $i = 6$

  The 6th eigenface is the **dominant difference** between the faces of person #37 and #38

〈 Person #37 〉



〈 Person #38 〉

〈 The 6th dominant eigenface 〉

(placement of the eyebrows)

Analysis and Conclusion

- 2282 column vectors in 32256 dimension

  -> reconstruction was generally successful when **r > 1000**

- Face recognition: 다양한 방법으로 coefficient를 비교할 수 있지만 여기서 사용한 방법은 두 사람의 coefficient의 **isolation**이 이루어지는지 확인함.
  - 두 사람 중 어떤 사람인지 모르는 이미지가 주어졌을 때 같은 방법으로 plot한 뒤 위치에 따라 판단 가능.
  - 여기선 coefficient 두개만 선택하여 2차원 평면 위에서 비교했지만, 3차원 또는 그 이상으로도 확장 가능.
  - 공통적으로 dominant하거나 minor한 eigenface의 계수로는 유의미한 분리가 이루어지지 않음.
  - 어떤 dataset을 이용하는지에 따라 비중 있게 다뤄야 할 coefficient가 변할 수 있음.

    Ex) 사람에 따라, 그림자에 따라, 이미지가 crop된 형태에 따라, etc.